

# Spatiotemporal Analysis of Solar Radiation for Sustainable Research in the Presence of Uncertain Measurements

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**Abstract.** The study of incoming solar radiation is important in the research for sustainable energy resources. Accurate description of the radiation spatiotemporal distribution is a key component in this research. To this end, the present work employs specialized tools from spatiotemporal stochastic analysis that enable us to account for the significant measurement uncertainty in the radiation observations. Specifically, we combine the global solar irradiance random field characteristics and available databases of global irradiance with the unique features of the Bayesian Maximum Entropy theory, and we obtain complete stochastic descriptions of the monthly-averaged daily global solar irradiance at selected locations across the United States for each month in 1999. The analysis produces global irradiance maps that can be further used by energy experts in decision-making and feasibility studies.

## 1 INTRODUCTION

Solar energy is one of the fields in the front line of research in renewable energy resources. Developments in the photovoltaics industry have transformed solar energy into a more viable solution in the last decade, and political and economic reasons are expected to increasingly lead consumers to solar energy for their needs in the future ([1]).

A key to the optimal utilization of this free and abundant source of energy lies in the understanding of the incoming solar radiation (irradiance) and its distribution across space and time. Typically, solar radiation measurements might not be directly available; consequently, similar analyses often compute radiation as a function of secondary meteorological observations, radiation models or satellite-measured quantities (e.g., [2]–[4]). Among different methodologies, tools from classical geostatistics have used the preceding resources as starting points for the study of solar radiation (e.g., [5], [6]). This work differs from the classical approaches in two aspects: First, we propose and employ nonlinear stochastic analysis methodology to interpolate and predict solar radiation across the USA by using terrestrial observations from a network of stations. Second, our computations account for the well-documented uncertainty in the input observations, as explained in the following section.

## 2 SOLAR RADIATION INFORMATION

Information used for solar radiation analysis is known to carry considerable uncertainty: Bland ([7]) reports a working level of 10% expected error for satellite estimates, and a minimum of 5% uncertainty for silicon cell pyranometers. Our analysis proposes a novel approach for the study of solar radiation by using the recorded ground observations and accounting for the reported data uncertainty to yield predictions of the global solar irradiance distribution across the USA for each month in 1999.

Global solar irradiance (GSI, measured in  $\text{MJm}^{-2}\text{d}^{-1}$ ) is the total amount of direct and diffuse solar radiation that is received on a horizontal surface on the ground. There is a relatively limited amount of historical data for GSI measurements in the USA. The US National Solar Radiation Database (NSRDB) has a significant amount of modeled solar radiation values for more than 1400 stations across the country. Still, only 40 stations have records of measured solar data, and none of them has a complete period of record.

This work considers the records of the 34 stations that have solar measurements in 1999. Their locations are shown on the left plot of Figure 1. Hourly (60-minute total) measurements of GSI are available at each station for most days in the year. NSRDB specifies a level of  $\pm 6\%$  uncertainty for the measured data ([8]). Given these initial hourly distributions at each spatiotemporal location, we produce daily totals of GSI. The daily GSI variance accounts for the hourly uncertainty as well as for correlation among the hourly values. Following this step, we compute the monthly average for the corresponding month in the year, and use these averages and their associated uncertainty as input for the analysis. Hence, we have soft probabilistic input data of monthly-averaged daily GSI measurements; in particular, each observation follows a Gaussian distribution with its own mean and variance. For example, the April 1999 probabilistic data means are displayed on the left-hand side plot of Figure 1.

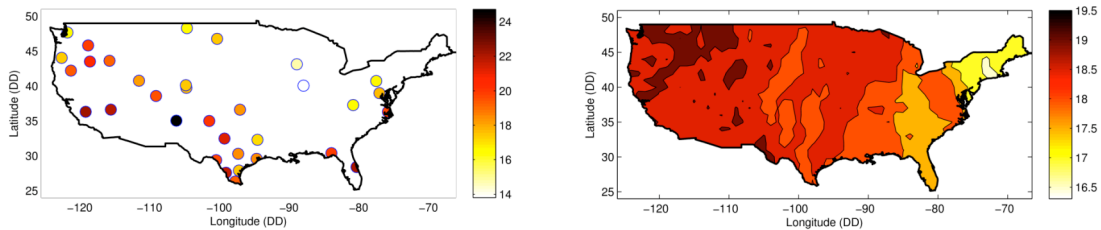


Figure 1: Left: Locations of observation stations and colorplot of monthly-averaged daily global solar irradiance (mdGSI) observations for April 1999. Right: mdGSI surface trend across the USA for April 1999. GSI values are expressed in  $\text{MJm}^{-2}\text{d}^{-1}$ .

## 3 METHODOLOGY AND ANALYSIS

The monthly-averaged daily GSI (mdGSI) distribution is represented as a spatiotemporal random field  $X_p$  (S/TRF; [9]).  $X_p$  consists of random variables  $x(\mathbf{p})$  at each space-

time point  $\mathbf{p} = (s, t)$  in the continuum of spatial coordinates  $\mathbf{s} = (s_1, s_2)$  and time  $t$ . We use the Bayesian Maximum Entropy (BME) method ([10]) that operates within a stochastic framework to incorporate and process the solar radiation input. The BME method follows a two-stage processing: First, we import general knowledge bases related to the natural process, and these lead to a prior probability density function (pdf)  $f_{\mathcal{G}}$ . Then,  $f_{\mathcal{G}}$  is updated with specificatory information from observed hard and soft measurements to produce the prediction pdf  $f_{\mathcal{X}}$ .

Our general knowledge consists of the data-based means and covariances of the mdGSI S/TRF. Knowledge of the S/TRF means allows us to perform predictions in a zero-mean S/TRF that is free of systematic spatiotemporal behavior. The only case-specific information is the soft probabilistic mdGSI data. Note that our case resembles the ordinary kriging (OK) mode of analysis, but differs fundamentally in that OK can only process hard data, i.e., observations with negligible or no associated uncertainty.

We start by removing a surface trend component from  $X_p$  by means of a nonlinear space/time moving window. For example, the right-hand side plot in Figure 1 shows the trend component of the mdGSI S/TRF for April 1999. The trend estimates are based on the soft data means, and we also consider the uncertainty of these estimates to have a negligible effect on the posterior distributions. See the work by Orton and Lark ([11]) on BME trend estimations for significant insight about these approximations.

Subsequently, we estimate the empirical covariance of the S/TRF from the detrended mdGSI residuals to investigate the form of underlying spatiotemporal correlations in the mdGSI S/TRF. A permissible covariance model is visually fit to the estimated covariance to describe the spatiotemporal correlation pattern the mdGSI field. Such models can comprise simple, space/time separable components, as well as space/time nonseparable functions based on a variety of physical considerations ([12]). Spatiotemporal correlation for the residual mdGSI is described by the space/time separable nested model in Table 1. Figure 2 displays a joint plot of the selected function superimposed on the empirical covariance estimates.

Table 1: Spatiotemporal covariance model used in the study. The sill expresses normalized variance.

Form (Spatial/Temporal)	Sill	Spatial range (DD)	Temporal range (Months)
Exponential/Exponential	0.35	1.5	8
Gaussian/Spherical	0.65	17.5	4.8

The versatile SEKS-GUI software framework ([13]) is used to perform BME-based predictions of the mdGSI distribution on a regular spatiotemporal grid across the USA. The spatial resolution is 1 decimal degree (DD) in both directions, and the temporal resolution is 1 month. The prediction grid has  $59 \times 26$  nodes in space and 12 nodes in time, thus giving a total of 18408 prediction locations.

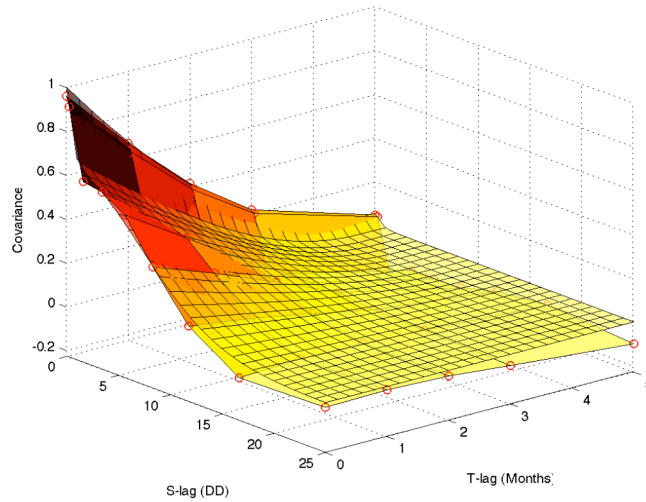


Figure 2: Spatiotemporal covariance model (densely gridded surface) used for the monthly-averaged daily global solar irradiance (mdGSI) predictions. The coarser grid connects the empirical covariance estimates (small circles at all coarse grid nodes). Note that only part of the empirical estimate circles are clearly visible because most of the coarser grid nodes lie underneath the theoretical model surface.

BME produces complete distributions of mdGSI by computing the mdGSI posterior probability density function (pdf) at each prediction spatiotemporal location; after the removed trends are re-established, the results are suitable for a broad spectrum of further analyses (e.g., management, efficiency, cost description, solar engineering assessment, etc.) Figure 3 shows the BME mode of the mdGSI posterior pdf across USA for four months in 1999. The patterns reveal the distribution of solar radiation in the different country regions during the year. The maps exhibit that the US southwest is overall a favorable region to develop solar energy projects, whereas the US mideastern states tend to receive comparatively less solar radiation.

Figure 4 has the prediction standard errors for the corresponding months in 1999 that appear in Figure 3. Note the geographic distribution of the error patterns in the figure. Apart from plausibly increased errors at grid nodes farther away from observation locations, larger errors also appear to occur at regions and times in the year where there are typically reduced clear sky conditions. Uncertainty in the daily averaged measurements can be influenced by meteorological conditions that affect the daily sums of the hourly observations, and Figure 4 shows how this effect is passed on to the prediction error.

## 4 DISCUSSION

A spatiotemporal stochastic analysis of the monthly-averaged daily GSI distribution is presented in this work. BME is the methodology of choice to account for the probabilistic observations data set, since it is uniquely known to incorporate uncertain data information in a rigorous manner among other prediction techniques.

The current study produces informative maps of predicted monthly-averaged global solar irradiance across the USA for 1999. The BME spatiotemporal predictions bring to the forefront the areas with increased GSI throughout the year. The findings indicate in a continental scale regions of feasibility for development of solar energy projects. In terms of finance and energy policy, these maps can be used in conjunction with the current variable energy pricing in different locations to suggest energy cost ratios. The combined information can feed the development of rational investment plans to benefit end-users (residential consumers in a centralized or distributed electricity grid) as well as commercial electricity consumers and producers.

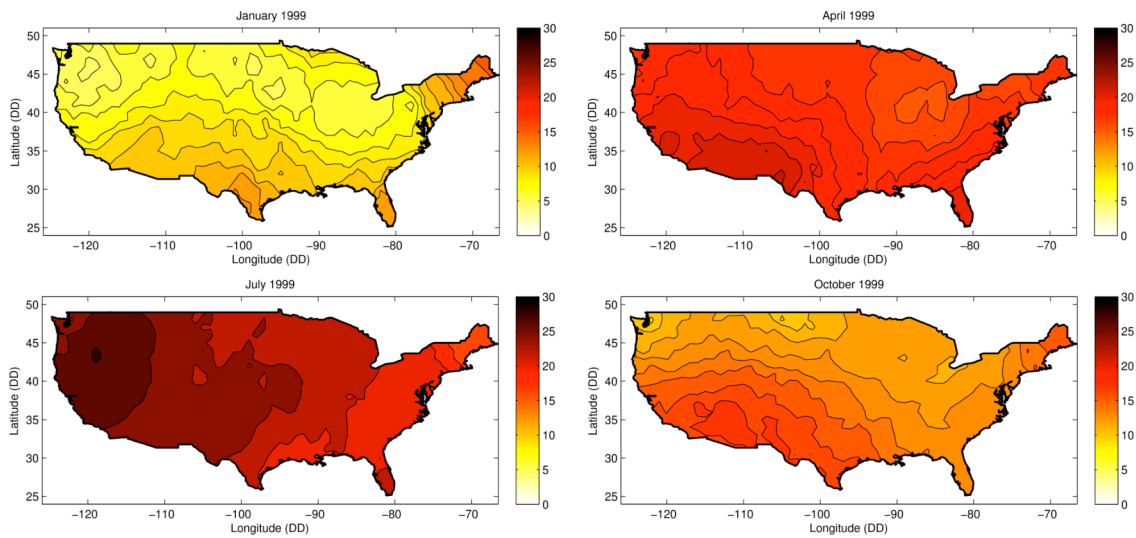


Figure 3: BME mode of mdGSI predictions on the USA grid at selected temporal instances in January, April, July, and October 1999. GSI is expressed in  $\text{MJm}^{-2}\text{d}^{-1}$  and presented in the same scale across all plots.

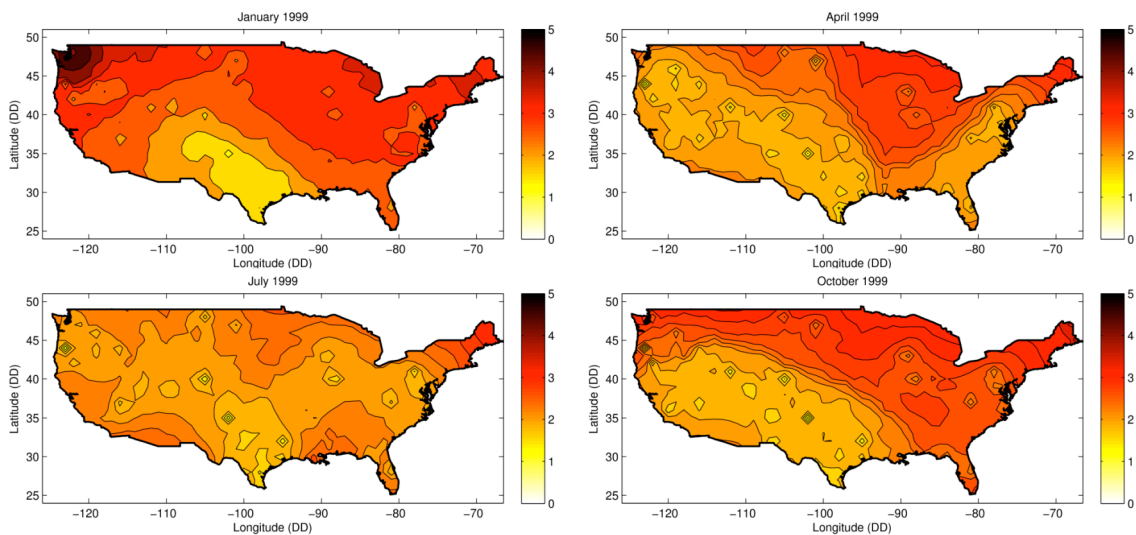


Figure 4: BME prediction standard error of the mdGSI predictions on the USA grid at selected temporal instances in January, April, July, and October 1999. The GSI standard error is expressed in  $\text{MJm}^{-2}\text{d}^{-1}$  and presented in the same scale across all plots.

## 5 REFERENCES

- [1] T. Bradford. Solar Revolution. The Economic Transformation of the Global Energy Industry. MIT Press, Cambridge, MA, 2006.
- [2] F. J. Batlles, M. A. Rubio, *et al.* Empirical Modeling of Hourly Direct Irradiance by Means of Hourly Global Irradiance. *Energy*, 25: 675-688, 1998.
- [3] H. F. Tovar, and J. M. Baldasano. Solar Radiation Mapping from NOAA AVHRR Data in Catalonia, Spain. *Journal of Applied Meteorology*, 40:1821-1844, 2001.
- [4] G. López, F. J. Batlles, and J. Tovar-Pescador. Selection of Input Parameters to Model Direct Solar Irradiance by Using Artificial Neural Networks. *Energy*, 30: 1675-1684, 2005.
- [5] V. D'Agostino, and A. Zelenka. Supplementing Solar Radiation Network Data by Co-Kriging with Satellite Images. *International J. of Climatology*, 12:749-761, 1992.
- [6] C. Ertekin, and F. Evrendilek. Spatio-temporal modeling of Global Solar Radiation Dynamics as a Function of Sunshine Duration for Turkey. *Agricultural and Forest Meteorology*, 145:36-47, 2007.
- [7] W. L. Bland. Uncertainty of Daily Insolation Estimates from a Mesoscale Pyranometer Network. *Journal of Atmospheric and Oceanic Technology*, 13:255-261, 1995.
- [8] NREL. National Solar Radiation Database 1991-2005 Update: User's Manual. Technical Report. National Renewable Energy Laboratory, Golden, CO, 2007.
- [9] G. Christakos. Random Field Models in Earth Sciences. Academic Press, Inc., San Diego, CA, 1992.
- [10] G. Christakos. Modern Spatiotemporal Geostatistics. Oxford University Press, New York, NY, 2000.
- [11] T. G. Orton, and R. M. Lark. Estimating the Local Mean for Bayesian Maximum Entropy by Generalised Least Squares and Maximum Likelihood, and an Application to the Spatial Analysis of a Censored Soil Variable. *European Journal of Soil Science*, 58:60-73, 2006.
- [12] A. Kolovos, G. Christakos, *et al.* Methods for Generating Non-Separable Spatio-temporal Covariance Models With Potential Environmental Applications. *Advances in Water Resources*, 27:815-830, 2004.
- [13] A. Kolovos, H.-L. Yu, and G. Christakos. SEKS-GUI v.0.6 User Manual. Department of Geography. San Diego State University, San Diego, CA, 2006.